L'Hopital's Rule

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Source: MacTutor

Introduction

In mathematics, finding limits is one of the most useful fundamentals in calculus. When taking the limit of a function, the desired result should be defined. When the limit result is undefined or in indeterminant form, L'Hopital's technique can be used to solve these indeterminant limits.

What is L'Hopital's Rule?

- Limits in indeterminate form did not have a solution.
- L'Hopital discovered that by taking the derivative of the numerator and denominator, this would resolve the indeterminate solution.
- Only used for $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

Nonapplicable Situations

- Limit is not indeterminant.
- Derivative of *f(x)* is defined, and *g(x)* is zero.
- Limit of derivative does not exist.

Uses of L'Hopital's Rule

- Proving Gamma Function

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$$

- Proving Compounding Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Examples

Applied Technique:

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2} = \frac{\sqrt{1+0} - 1 - \frac{0}{2}}{0^2} = \frac{\sqrt{1} - 1 - 0}{0} = \frac{0}{0}$$

$$f(x) = \sqrt{1+x} - 1 - \frac{x}{2}, \qquad g(x) = x^2$$

$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}(1) - 0 - \frac{1}{2}, \; g'(x) = 2x$$

$$\lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}}{2x} = \frac{\frac{1}{2}(1+0)^{-\frac{1}{2}} - \frac{1}{2}}{2(0)} = \frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0}$$

$$f(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}, \qquad g(x) = 2x$$

$$f'(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} - 0, \; g'(x) = 2$$

$$\lim_{x\to 0} \frac{-\frac{1}{4}(1+x)^{-\frac{3}{2}}}{2} = \frac{-\frac{1}{4}(1+0)^{-\frac{3}{2}}}{2} = \frac{-\frac{1}{4}(1)}{2} = -\mathbf{\frac{1}{8}}$$

Nonapplicable Situation:

$$\lim_{x\to 0} \frac{x^3 + 5}{3x^2 + x} = \frac{0^3 + 5}{3(0)^2 + 0} = \frac{5}{0} = \textbf{Undefined}$$

History

- Bernoulli worked under L"Hopital because of the potential that L'Hopital saw.
- He shared ideas that led to the creation of L'Hopital's rule.
- Since Bernoulli worked under L'Hopital, major discoveries by Bernoulli were published by L'Hopital and practically no credit was given to Bernoulli.
- Nowadays, Bernoulli is credited with an alternate naming of the technique, Bernoulli's rule.

Impact

L'Hopital's rule has become a staple concept in mathematics, specifically calculus. The simplification of the evaluation of indeterminant limits revolutionized mathematics. Limits became easier to solve and their use became more widespread. Through its complicated history behind the naming of the technique, all contributors ended up credited. Bernoulli played a major role in helping L'Hopital derive this technique.

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