The Russell's Paradox

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What is Russell's Paradox?

- Consider the property of sets of not being members of themselves.
- If the property determines a set, call it A, then A is a member of itself if and only if A is not a member of itself.
- $Let\ x\ \in x,\ and\ let\ R = \{x: \sim\emptyset(x)\}$, then R is the set with members that are not members of themselves.

Figure 1: barbershop paradox

Any easier way to understand the paradox?

- The early-on **barbershop paradox** by Lewis Carroll.
- Barber Paradox:
  - Assume there is a barber who claims to shave some kind of people--only those that do not shave themselves.
  - The contradiction arose while asking if the barber shaves himself.
  - If yes, there's a contradiction that he did shave people who shave themselves.
  - If no, then there is also a contradiction that he didn't shave those who do not shave themselves.

Solution Approach

- Russell initialed his own response to his paradox called ***theory of types***.
- Russell's response motivated the development of ***vicious circle principle***.
- From 1908, Zermelo axiomatization work has pushed the development of basic set-theoretic principles, known as ***Zermelo-Fraenkel Set Theory***.
- Later on, John Von Neumann introduced the axiom of regularity (axiom of foundation). The Zermelo-Fraenkel axioms plus the Axiom of Choice (**ZFC**) led the development of mathematics to the standard axiom system of set theory.

History

- In 1897, Cesare Burali-Forti had similar paradox which demonstrates the antinomy in the set with elements of all ordinal numbers.
- The Burali-Forti paradox contradicted a proof by mathematician Cantor.
- Between 1897 and 1902, Zermelo had a forerunner paradox, as one of a cluster argument, and anticipated Russell's Paradox for many years.
- Zermelo's paradox created the problem of inconsistency to Gottlob Frege's theory.
- In 1902, Russell included the discovery of his paradox in a letter to Frege, and this letter was in press as the second volume of Frege's book.
- With the communication with Frege, Russell began to write the Appendix B for his soon-to-be-released book, *Principles of Mathematics*, and this appendix soon became the famous "Russell's paradox."

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