Pascal's Triangle

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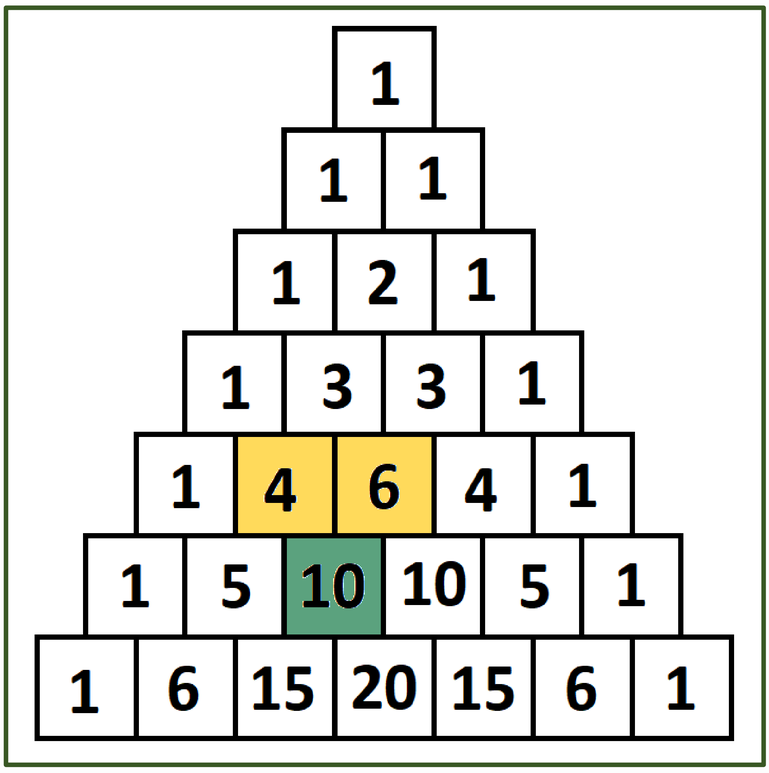
What is Pascal's Triangle?

- Pascal's Triangle is a triangular array of numbers with many surprising properties
- To construct Pascal's Triangle:
  - Start with a row containing only 1
  - Add more rows, each offset slightly from the previous one
  - Entries in each row are generated by adding the 2 entries above it, imagining a 0 where there is a blank space



History

- Pascal's Triangle has a rich and complex history
- Pascal's Triangle goes by many names, as it was discovered independently by many people
- Often called:
  - *Yang Hui's Triangle* in the Far East (after the 13th century Chinese mathematician)
  - *Khayyam's Triangle* in the Middle East (after 11th century Arab mathematician Omar Khayyam)
- However, even in these regions, the triangle was likely studied centuries before their eponyms
- It now bears 17th century French mathematician Blaise Pascal's name due to his revolutionary publication *Traité du Triangle Arithmétique* (or *Treatise on the Arithmetical Triangle*)
- Pascal was the first to prove many properties about the triangle

Other Ways to Make the Triangle

- The entries in Pascal's Triangle are called *binomial coefficients*, since they are the coefficients upon expansion of $(a+b)^n$
- For example, expanding $(a+b)^3$, we get:

$$\mathbf{1}a^3 + \mathbf{3}a^2b + \mathbf{3}ab^2 + \mathbf{1}b^3$$

- Alternatively, starting at 0, each entry in the triangle is the number of different ways you can choose r objects from n, read $n\ choose\ r$
- For example, given 5 objects, the number of ways you can pick 2 of them is $5\ choose\ 2 = 10$

Properties

- Summing along each row gives powers of 2. For example,

$$1 + 4 + 6 + 4 = 16 = 2^4$$

- Concatenating each row gives powers of 11. For example,

$$14641 = 11^4$$

  - Carry as needed for entries with more than 1 digit
- *Many* more properties and pattern, some extremely deep and surprising, exist in Pascal's Triangle
- Even some of the superstars of math, like constants π, e, and the golden ratio ϕ, and Sierpinski's Triangle can be found
- Can you find any more patterns in Pascal's Triangle?

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