Gabriel Cramer (July 31st, 1704 – January 4th, 1752)

CRAMER'S RULE

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Introduction

- *Cramer's Rule is a method for solving linear simultaneous equations. It makes use of determinants, thus a knowledge of determinants is necessary.*
- *To use Cramer's Rule, the system must have as many equations as there are unknowns.*

Coefficient Matrices

- *You can use determinants to solve a system of linear equations.*
- *You use the coefficient matrix of the linear system.*
- **Linear System** / **Coeff Matrix**

ax + by = e
cx + dy = f

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Key Points

- *The denominator consists of the coefficients of variables (x in the first column, and y in the second column).*
- *The numerator is the same as the denominator, with the constants replacing the coefficients of the variable for which you are solving.*

Applying Cramer's Rule on a System of Two Equations

$$\begin{cases} ax+by=e \\ cx+dy=f \end{cases}$$

$$D=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$D_x=\begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

$$D_y=\begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

$$x=\frac{D_x}{D} \quad y=\frac{D_y}{D}$$

$$\begin{cases} 2x-3y=-16 \\ 3x+5y=14 \end{cases}$$

$$D=\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix}=(2)(5)-(-3)(3)=10+9=19$$

$$D_x=\begin{vmatrix} -16 & -3 \\ 14 & 5 \end{vmatrix}=(-16)(5)-(-3)(14)=-80+42=-38$$

$$D_y=\begin{vmatrix} 2 & -16 \\ 3 & 14 \end{vmatrix}=(2)(14)-(3)(-16)=28+48=76$$

$$x=\frac{D_x}{D}=\frac{-38}{19}=-2 \qquad y=\frac{D_y}{D}=\frac{76}{19}=4$$

Using Cramer's Rule to Solve a System of Three Equations

Consider the following equations:

$$2x_1-4x_2+5x_3=36$$
$$-3x_1+5x_2+7x_3=7$$
$$5x_1+3x_2-8x_3=-31$$
$$[A][x]=[B]$$

where

$$[A]=\begin{bmatrix} 2 & -4 & 5 \\ -3 & 5 & 7 \\ 5 & 3 & -8 \end{bmatrix} \qquad [x]=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } [B]=\begin{bmatrix} 36 \\ 7 \\ -31 \end{bmatrix}$$

$$D=\begin{vmatrix} 2 & -4 & 5 \\ -3 & 5 & 7 \\ 5 & 3 & -8 \end{vmatrix}=-336$$

$$D_1=\begin{vmatrix} 36 & -4 & 5 \\ 7 & 5 & 7 \\ -31 & 3 & -8 \end{vmatrix}=-672 \qquad D_2=\begin{vmatrix} 2 & 36 & 5 \\ -3 & 7 & 7 \\ 5 & -31 & -8 \end{vmatrix}=1008$$

$$D_3=\begin{vmatrix} 2 & -4 & 36 \\ -3 & 5 & 7 \\ 5 & 3 & -31 \end{vmatrix}=-1344$$

$$x_1=\frac{D_1}{D}=\frac{-672}{-336}=2$$
$$x_2=\frac{D_2}{D}=\frac{1008}{-336}=-3$$
$$x_3=\frac{D_3}{D}=\frac{-1344}{-336}=4$$

Conclusion

- *Not all systems have a definite solution. If the determinant of the coefficient matrix is zero, a solution cannot be found using Cramer's Rule due to this causing division by zero.*

  When the solution cannot be determined, one of two conditions exists:
  - *The planes graphed by each equation are parallel and there are no solutions.*
  - *The three planes share one line (like three pages of a book share the same spine) or represent the same plane, in which case there are infinite solutions.*

References

1. Burton, D. M. (2007). The history of mathematics: An introduction. New York: McGraw-Hill.
2. Larson, R. (2017). Elementary linear algebra: Metric version.
3. Mesacc.edu. 2020. *Using Cramer'S Rule To Solve Three Equations With Three Unknowns*. [online] Available at: <http://www.mesacc.edu/~scotz47781/mat150/notes/cramers_rule/Cramers_Rule_3_by_3_Notes.pdf> [Accessed 15 February 2020].