

# The Four - Color Problem

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## Question

What is the minimum amount of colors it takes to color in a map where no adjacent regions share the same color?



Figure 1: map of the united states

## Relation to Graph Theory

- Map = network where each region is a vertex every adjacent region is connected by an edge
- All networks must be planar in order to represent a map

## History

- Question first asked in 1852 by Francis Guthrie
- 1879 Alfred Bray Kempe introduced Kempe Method and published a proof for the problem
- Heawood disproved Kempe's proof and showed that if the number of edges around each region is divisible by 3, then the regions are four-colorable.
- 1976 Kenneth Appel and Wolfgang Haken found a solution

## Controversy

- If map is imposed with certain restrictions the theorem does not hold true
- Problem was one of the first to be solved with a computer program



Figure 2: Coloring map with specifications can require more than 4 colors

## Important Concept

### Kempe Chains

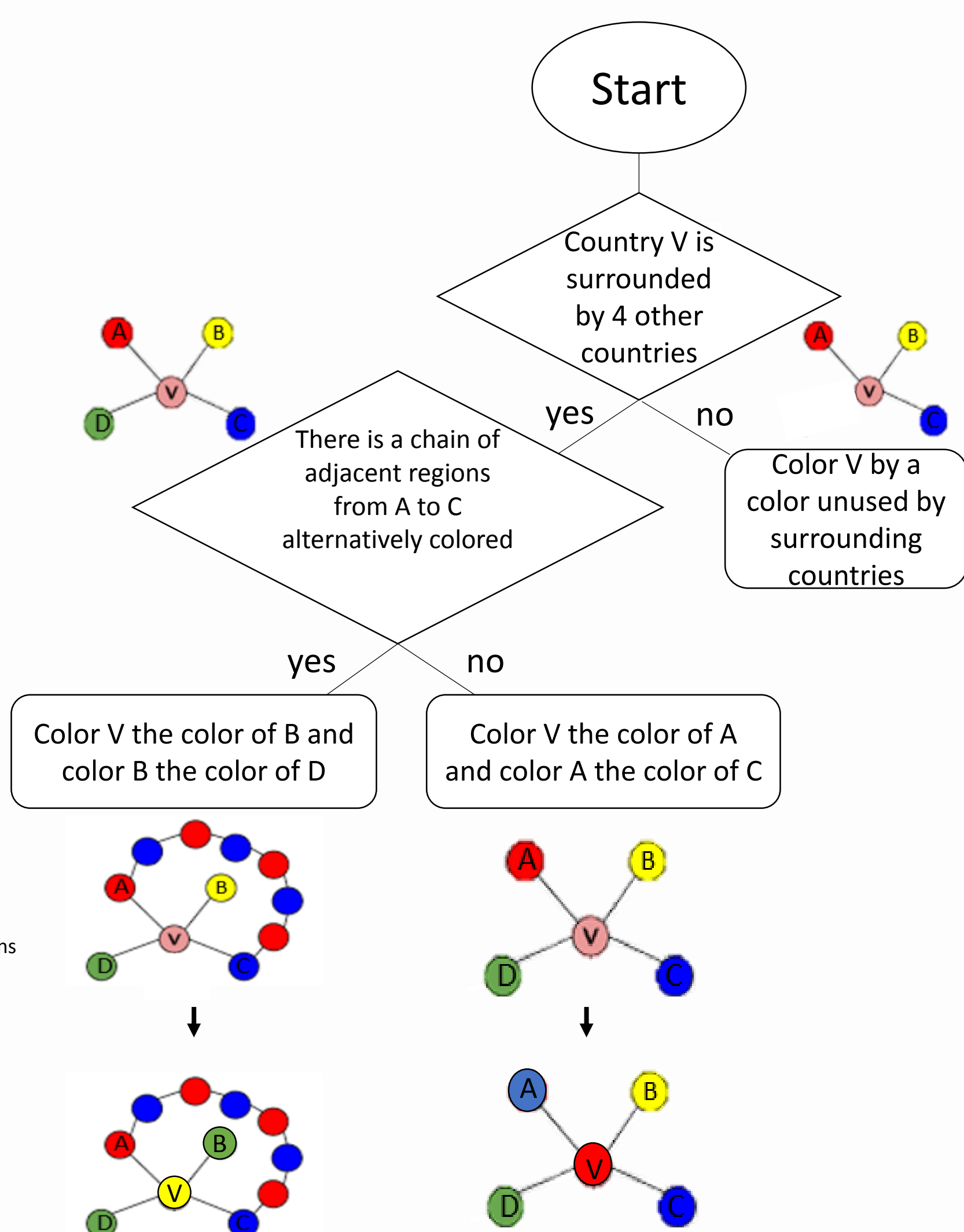


Figure 3: Network used to represent Kempe Chains

## References:

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- Maritz, P., & Mouton, S. (1999). The Mathematical Tourist. *Francis Guthrie: A Colorful Life*, 21(3), 67–75. doi: 10.1007/bf03025412
- Figure 1. Robertson, N., Sanders, D. P., & Thomas, R. (2017). *Map of the United States*. Retrieved from <https://people.math.gatech.edu/~thomas/FC/fourcolor.html>
- Figure 2. Wirth, E. (2019, February 21). Four Color Theorem Controversy. Retrieved January 28, 2020, from <https://mathsection.com/four-color-theorem-controversy/?cookie-state-change=1580389726045>
- Figure 3. Kempe Chain.png. (2020, February 29). *Wikimedia Commons, the free media repository*. Retrieved 14:30, March 16, 2020 from [https://commons.wikimedia.org/w/index.php?title=File:Kempe\\_Chain.png&oldid=400061942](https://commons.wikimedia.org/w/index.php?title=File:Kempe_Chain.png&oldid=400061942).