L'Hôpital's Rule

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History

- Method discovered by Johann Bernoulli
- Guillaume de L'Hôpital developed this method into L'Hôpital's Rule
- Published in the first textbook on differential calculus

Definition

Suppose *f* and *g* are differentiable functions such that

1. *g'(x) =0* on an open interval *I* containing *a;*
2. $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, or $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$;
3. $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists.

Then, $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

Indeterminate Forms

Apply the rule

$$\frac{0}{0}, \frac{\infty}{\infty}$$

Rewrite and then apply

$$\infty - \infty, 0(\infty),$$
$$0^0, 1^\infty, \infty^0$$

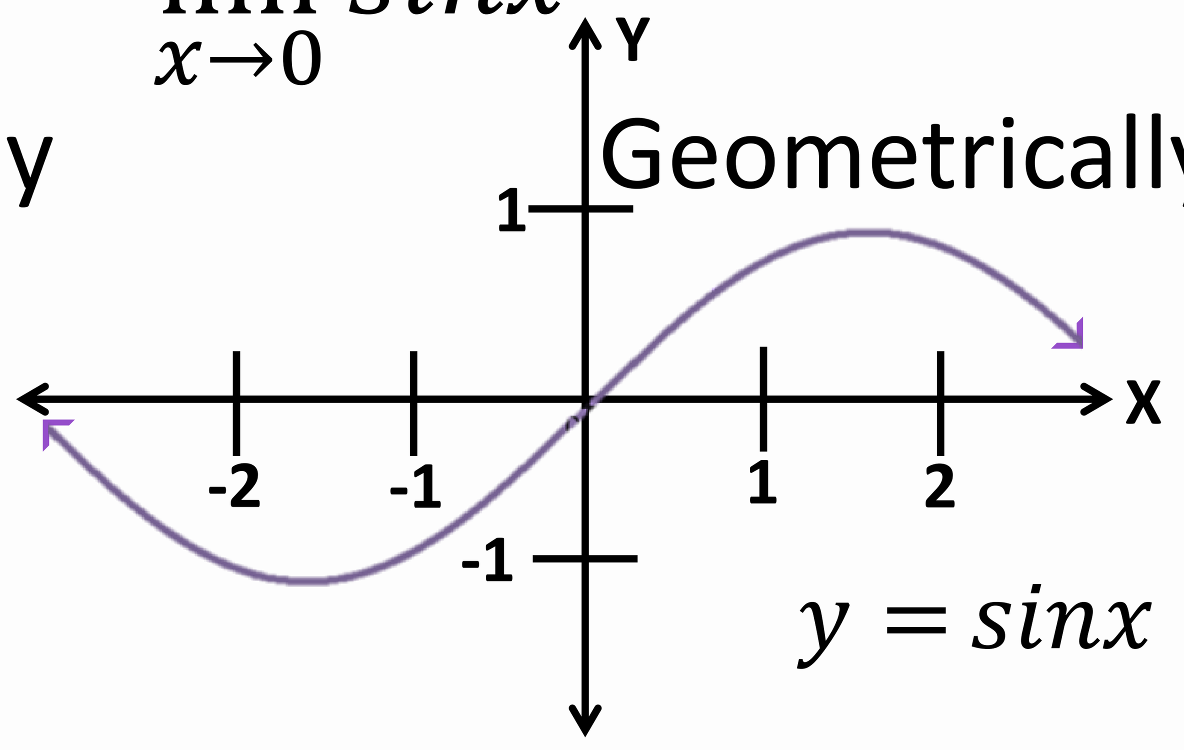
Solving a Limit

$$\lim_{x\to 0} sinx$$

Algebraically

$$\lim_{x\to 0} sinx$$
$$= \sin 0$$
$$= 0$$

Geometrically

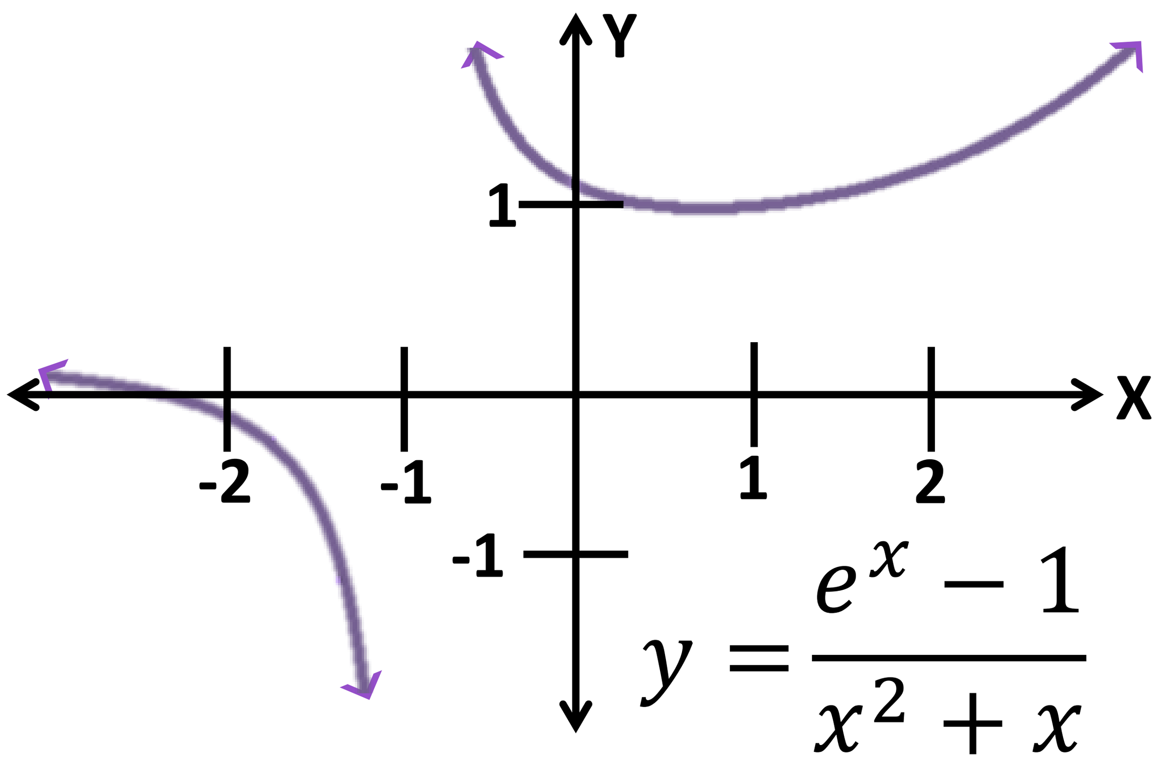


Example

$$\lim_{x\to 0}\frac{e^x - 1}{x^2 + x} = \frac{0}{0}$$

Apply L'Hôpital's Rule

$$\lim_{x\to 0}\frac{e^x-1}{x^2+x} = \lim_{x\to 0}\frac{\frac{d}{dx}(e^x-1)}{\frac{d}{dx}(x^2+x)}$$
$$= \lim_{x\to 0}\frac{e^x}{2x+1} = 1.$$



Example

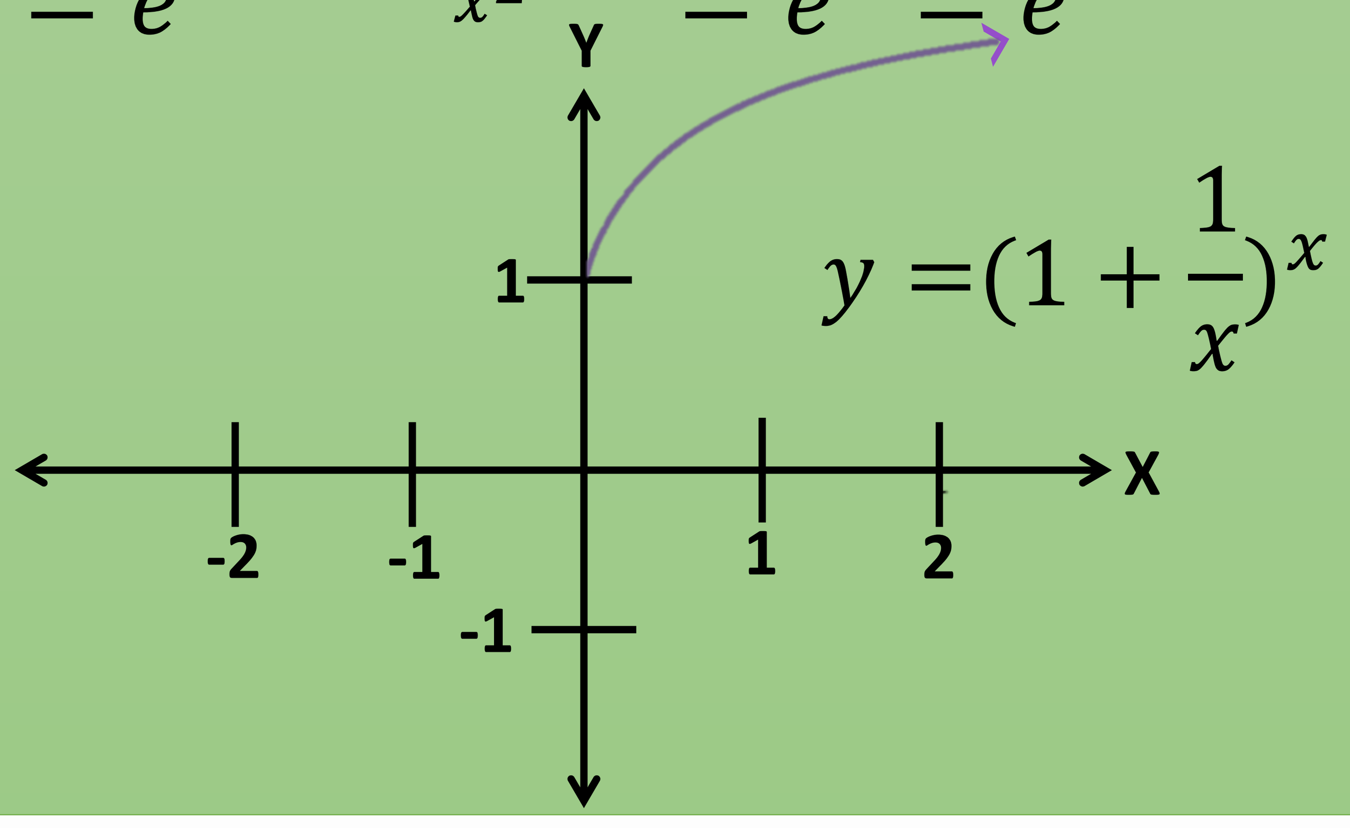
$$\lim_{x\to\infty}(1+\frac{1}{x})^x = 1^\infty$$

Rewrite

$$\lim_{x\to\infty}(1+\frac{1}{x})^x = e^{\ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right)}$$
$$= e^{\left(\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^x\right)} = e^{\lim_{x\to\infty} x\ln(1+\frac{1}{x})}$$
$$= e^{\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}} = \frac{0}{0}$$

Apply L'Hôpital's Rule

$$e^{\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}} = e^{\lim_{x\to\infty}\frac{\frac{d}{dx}(\ln\left(1+\frac{1}{x}\right))}{\frac{d}{dx}(\frac{1}{x})}}$$
$$= e^{\lim_{x\to\infty}\frac{(\frac{1}{1+\frac{1}{x}})(\frac{-1}{x^2})}{(\frac{-1}{x^2})}} = e^1 = e$$



Resources

- Goh, Pl Han, et al. "L'Hôpital's Rule." *Brilliant Math & Science Wiki*, brilliant.org/wiki/lhopitals-rule/
- Hughes-Hallett, Deborah, editor. Calculus. 4th ed, J. Wiley, 2005.
- Hosch, William. "L'Hôpital's Rule." Encyclopædia Britannica, Encyclopædia Britannica, Inc., 4 Aug. 2011, www.britannica.com/science/LHopitals-rule.